SVRE: NEW METHOD FOR TRAINING GANs

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GENERATIVE MODELING AND MODEL-BASED REASONING FOR ROBOTICS AND AI WORKSHOP

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Reducing Noise in GAN Training with Variance Reduced Extragradient

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* Equal contribution

Generative Adversarial Networks

[Goodfellow et al., 2014]

Challenges

- Standard supervised learning:

$$\min_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{\theta})$$

- GANs: ~~Hard~~ (different) optimization problem: minimax.

CHALLENGES

- Standard supervised learning:

$$\min_{\theta} \mathcal{L}(\theta)$$

- GANs: ~~Hard~~ (different) optimization problem: minimax.

$$\min_{\theta_G} \max_{\theta_D} V(\theta_G, \theta_D)$$

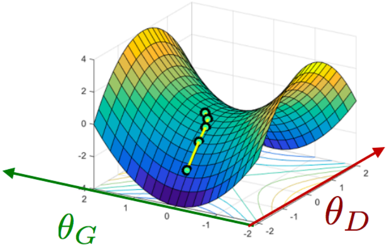
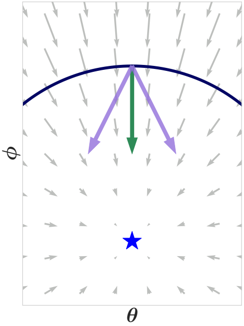


Image source: *Vaishnavh Nagarajan*

“Noise”: noisy gradient estimates
Due to stochasticity

- Using sub-samples (mini-batches) of the full dataset to update the parameters
- Variance Reduced (VR) Gradient: optimization methods that reduce such noise

Minimization: Single-objective

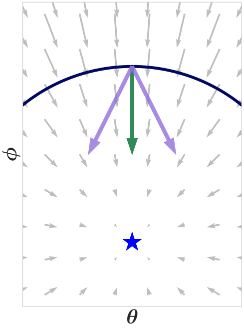


Batch method direction

Stochastic method direction: noisy

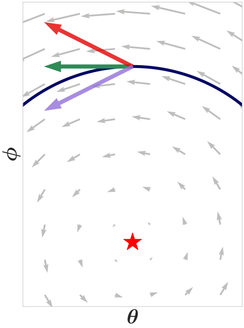
Variance Reduction–Motivation for Games

- Intuitively: **Minimization** *vs.* **Game** (Noise from Stochastic gradient)
- Empirically: **BigGAN**–"Increased batch size significantly improves performances"
- To sum up, two issues:



Minimization

"approximately" the right direction



Game

Direction with noise can be "bad"

Variance Reduction–Motivation for Games

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Brock et al. [2018] report a relative improvement of 46% of the Inception Score metric [Salimans et al., 2016] on **ImageNet** if the mini-batch size is increased 8–fold.

Variance Reduction–Motivation for Games

- Intuitively: **Minimization** *vs.* **Game** (Noise from Stochastic gradient)
- Empirically: **BigGAN**–"Increased batch size significantly improves performances"
- To sum up, two issues:

  - Adversarial aspect from min-max $\rightarrow$ Extragradient.
  - Noise from stochastic gradient $\rightarrow$ Variance Reduction.

Extragradient

Extragradient

Two players $\boldsymbol{\theta}$, $\boldsymbol{\varphi}$. Idea: perform a "Lookahead step"

$$\text{Extrapolation:} \begin{cases} \boldsymbol{\theta}_{t+1/2} = \boldsymbol{\theta}_t - \eta \nabla_{\boldsymbol{\theta}} \mathcal{L}_G(\boldsymbol{\theta}_t, \boldsymbol{\varphi}_t) \\ \boldsymbol{\varphi}_{t+1/2} = \boldsymbol{\varphi}_t - \eta \nabla_{\boldsymbol{\varphi}} \mathcal{L}_D(\boldsymbol{\theta}_t, \boldsymbol{\varphi}_t) \end{cases}$$

$$\text{Update:} \begin{cases} \boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t - \eta \nabla_{\boldsymbol{\theta}} \mathcal{L}_G(\boldsymbol{\theta}_{t+1/2}, \boldsymbol{\varphi}_{t+1/2}) \\ \boldsymbol{\varphi}_{t+1} = \boldsymbol{\varphi}_t - \eta \nabla_{\boldsymbol{\varphi}} \mathcal{L}_D(\boldsymbol{\theta}_{t+1/2}, \boldsymbol{\varphi}_{t+1/2}) \end{cases}$$

Variance Reduced Gradient Methods

Variance Reduced Estimate of the Gradient

Based on Finite sum assumption:

$$\frac{1}{n}\sum_{i=1}^{n}\mathcal{L}(\boldsymbol{x}_i,\boldsymbol{\omega}),$$

Epoch based algorithm:

- Save the full gradient $\frac{1}{n}\sum_i \nabla\mathcal{L}(\boldsymbol{x}_i,\boldsymbol{\omega}^{\mathcal{S}})$ and the snapshot $\boldsymbol{\omega}^{\mathcal{S}}$.
- For one epoch use the update rule:

$$\boldsymbol{\omega} \leftarrow \boldsymbol{\omega} - \eta\Big[\underbrace{\nabla\mathcal{L}(\boldsymbol{x}_i,\boldsymbol{\omega})}_{\text{Stochastic gradient}} + \underbrace{\frac{1}{n}\sum_i \nabla\mathcal{L}(\boldsymbol{x}_i,\boldsymbol{\omega}^{\mathcal{S}}) - \nabla\mathcal{L}\left(\boldsymbol{x}_i,\boldsymbol{\omega}^{\mathcal{S}}\right)}_{\text{correction using saved past iterate}}\Big]$$

- Requires 2 stochastic gradients (at the current point and at the snapshot).
- If $\boldsymbol{\omega}^{\mathcal{S}}$ is close to $\boldsymbol{\omega}$ $\rightarrow$ close to full batch gradient $\rightarrow$ small variance.
- Full batch gradient expensive but tractable, *e.g.*, compute it once per pass.

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SVRE: Variance reduction + Extragradient

Pseudo–algorithm

1. Save snapshot $\boldsymbol{\omega}^{\mathcal{S}} \leftarrow \boldsymbol{\omega}_t$ and compute $\frac{1}{n}\sum_i \nabla\mathcal{L}(\boldsymbol{x}_i, \boldsymbol{\omega}^{\mathcal{S}})$.
2. For i in $1, \ldots, \texttt{epoch\_length}$:
   - Compute $\omega_{t+\frac{1}{2}}$ with variance reduced gradients at ω_t.
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   - $t \leftarrow t+1$
3. Repeat until convergence.

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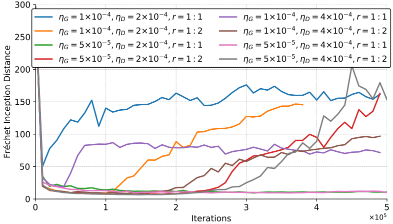
SVRE yields the fastest convergence rate for strongly convex stochastic game optimization in the literature.

SVRE: EXPERIMENTS

EXPERIMENTS
SVRE YIELDS STABLE GAN OPTIMIZATION

Stochastic baseline



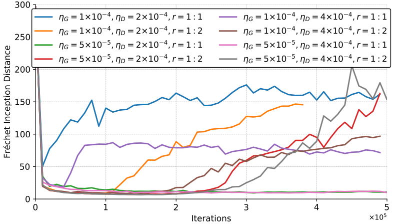
– Always diverges.

– Many hyperparameters ($\eta_G, \eta_D, \beta_1, \gamma, r$).

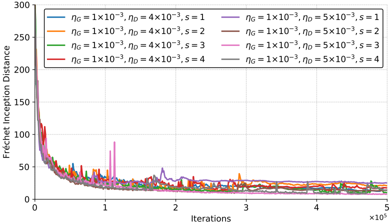
\+ if convergence $\rightarrow$ fast

Experiments
SVRE yields stable GAN optimization

Stochastic baseline



SVRE



– Always diverges.
– Many hyperparameters ($\eta_G, \eta_D, \beta_1, \gamma, r$).
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+ Does not diverge.
+ fewer hyperparameters (omits β_1, γ, r)
– slower for very deep nets.

SVRE: Takeaways

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- Controlling variance is more critical for games (could be reason behind success of *Adam* on GANs)
- SVRE: combines Extragradient and variance reduction.
- Best convergence rate (under some assumptions) for games.
- Good stability properties.

THANKS.

Questions?

References I

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Ian Goodfellow, Jean Pouget-Abadie, Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair, Aaron Courville, and Yoshua Bengio. Generative adversarial nets. In Advances in Neural Information Processing Systems 27, pages 2672–2680. 2014.

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Appendix

The GAN framework

Equilibrium at $p_g = p_d$

The discriminator maximizes:

$$
\begin{aligned}
V(G, D) &= \int_x p_d(x) \log(D(x)) \, \mathrm{d}x + \int_z p_z(z) \log(1 - D(G(z))) \, \mathrm{d}z \\
&= \int_x p_d(x) \log(D(x)) + p_g(x) \log(1 - D(x)) \, \mathrm{d}x
\end{aligned}
$$

Where we used $x = G(z)$, and p_g is the distribution of x.
Hence, the optimal discriminator D^* is:

$$\boxed{D^*(x) = \frac{p_d(x)}{p_d(x) + p_g(x)}}$$

The GAN framework

Equilibrium at $p_g = p_d$

The generator minimizes:

$$
\begin{aligned}
V(G, D^*) &= \mathop{\mathbb{E}}_{x\sim p_d}[\log D^*(x)] + \mathop{\mathbb{E}}_{x\sim p_g}[\log(1 - D^*(x))] \\
&= \mathop{\mathbb{E}}_{x\sim p_d}[\log \frac{p_d(x)}{p_d(x) + p_g(x)}] + \mathop{\mathbb{E}}_{x\sim p_g}[\log \frac{p_g(x)}{p_d(x) + p_g(x)}] \\
&= -\log 4 + \mathbb{D}_{KL}(p_d || \frac{p_d + p_g}{2}) + \mathbb{D}_{KL}(p_g || \frac{p_d + p_g}{2}) \\
&= -\log 4 + 2 \cdot \mathbb{D}_{JS}(p_d || p_g)
\end{aligned}
$$

where we used: $\mathbb{D}_{JS}(p\|q) = \frac{1}{2}\mathbb{D}_{KL}(p\|\frac{p+q}{2}) + \frac{1}{2}\mathbb{D}_{KL}(q\|\frac{p+q}{2})$.

The optimum is reached when $p_g = p_d$,
and the optimal value is $-\log 4$.